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A 3-D INVERSE PROBLEM IN ESTIMATING THE TIME-DEPENDENT HEAT TRANSFER COEFFICIENTS FOR PLATE FINS

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Abstract - The local time-dependent surface heat transfer coefficients for plate finned-tube heat exchangers are estimated in a three-dimensional inverse heat conduction problem. The inverse algorithm utilizing the Steepest Descent Method (SDM) and a general purpose commercial code CFX4.4 is applied successfully in this study in accordance with the simulated measured temperature distributions on the fin surface by infrared thermography. Two different heat transfer coefficients for staggered as well as in-line tube arrangements with different measurement errors are determined. Results of the numerical simulation show that the reliable estimated heat transfer coefficients can be obtained by using the present inverse algorithm.

1. INTRODUCTION

Heat exchangers are the workhorse of industry. Variously known as condensers, coolers, evaporators, heaters, vaporizers, and so forth. Finned surfaces of the plate finned-tube heat exchangers have been in use over a long period of time for dissipation of heat by convection. Applications for finned surfaces are widely seen in air-conditioning, electrical, chemical, refrigeration, cryogenics and many cooling systems in industry. Kays and London [9] introduced various types of heat transfer surfaces.

The estimation of the convective heat transfer coefficient is more difficult to perform than other common thermo-fluid-dynamic quantities, especially in the case of non-uniform distributions and/or of conduction-convection problems. Ay *et al*. [2] applied a control volume based finite difference formulation and an infrared thermography based temperature measurements to estimate the local heat transfer coefficients of a plate fin in a 2-D inverse heat conduction problem. Recently, Huang *et al*. [7] used the technique of Steepest Descent Method (SDM) and commercial code CFX4.4 [3] to estimate the local convective heat transfer coefficients over finned surfaces in a steady-state 3-D inverse heat conduction problem based on the simulated temperature measurements by infrared thermography. However the 3-D inverse heat conduction problem in estimating the time-dependent local convective heat transfer coefficients on finned surface has never been examined.

The technique of utilizing the inverse algorithms together with the commercial code CFX4.2 has been developed successfully by Huang and Wang [6], they applied the algorithm to estimate the unknown surface heat fluxes in a 3-D solid. By following a similar technique, Huang and Chen [4] estimated successfully the unknown boundary heat flux in a 3-D inverse heat convection problem. More recently, Huang and Li [5] applied the algorithm to an optimal heating problem in determining the optimal surface heat fluxes for a 3-D forced convection problem.

It should be noted that all of the above applications are 3-D inverse problems, this implies that the algorithm is powerful since the 3-D inverse problems are still very limited in the open literature.

The objective of this study is to extend a 3-D steady-state inverse problem [7] to a transient 3-D inverse problem in estimating the time-dependent local convective heat transfer coefficients of finned surfaces for the plate finned-tube heat exchangers. The number of unknown heat transfer coefficients will increase tremendously under the present consideration and this will also increase the difficulty in solving the present inverse problem.

2. DIRECT PROBLEM

A typical plate finned-tube heat exchanger is shown in Figure 1(a). The plate fins of staggered arrangement with domain $\Omega(x,y,z)$ is illustrated in Figure 1(b). The surfaces S_i, i = 1 to 6, are subjected to a convective boundary condition with prescribed heat transfer coefficient $h(S_i,t)$, i = 1 to 6, where i = 1 to 4 represent the edge boundaries; while i = 5 and 6 indicate the top and bottom surfaces, respectively. The unknown heat transfer coefficient $h(S_i,t)$ could be a function of the temperature in the present study. The tube boundary surfaces S_i, i = 7 to (I+6), are subjected to a prescribed temperature condition $T = T(S_i,t)$, where I represents the number of tubes.

The edge surface area S_i, i = 1 to 4 is small enough when comparing with top and bottom surfaces S_i, i = 5 to 6. This implies that the heat transfer rate through S_i, i = 1 to 4 can be neglected. For this reason we assume that the boundary conditions on surface S_i, i = 1 to 4 are adiabatic conditions. Meanwhile, since the fin thickness is thin, the temperature distribution on S_5 should be very close to S_6 for any time t, therefore it is also reasonable to assume that the heat transfer coefficients on S_5 and S_6 are equal to each other , i.e. $h(S_5,t) = h(S_6,t)$. The direct problem becomes

$$k[\frac{\partial^2 T(\Omega,t)}{\partial x^2}+\frac{\partial^2 T(\Omega,t)}{\partial y^2}+\frac{\partial^2 T(\Omega,t)}{\partial z^2}]=\rho Cp\frac{\partial T(\Omega,t)}{\partial t}\ ;\ \text{in } \Omega(x,y,z,t)\,,\ t>0 \tag{1a}$$

$$\frac{\partial T(S_i,t)}{\partial n}=0 \qquad ;\ \text{on fin surface } S_i\ \ i=1 \text{ to } 4,\ t>0 \tag{1b}$$

$$-k\frac{\partial T(S_5,t)}{\partial z}=h(S_5,t)(T_\infty - T)\ ;\ \text{on fin surface } S_5,\ t>0 \tag{1c}$$

$$-k\frac{\partial T(S_6,t)}{\partial z}=h(S_6,t)(T - T_\infty)\ ;\ \text{on fin surface } S_6\ ,\ t>0 \tag{1d}$$

$$T(S_i,t)=T_o \qquad ;\ \text{on tube surfaces, } i=7 \text{ to } I+6\ ,\ t>0 \tag{1e}$$

$$T(\Omega,t)=T_\infty \qquad ;\ \text{for } t=0 \tag{1f}$$

Here k is the thermal conductivity of the fin, ρ and Cp are the density and heat capacity of the material, respectively. The direct problem considered here is concerned with calculating the plate fin temperatures when the heat transfer coefficient $h(S_i,t)$, i = 5 and 6, thermal properties as well as the initial and boundary conditions on tube surfaces are known. The solution for the above 3-D heat conduction problem in domain Ω is solved using CFX4.4 and its Fortran subroutine USRBCS.

3. THE INVERSE PROBLEM

For the inverse problem considered here, the local time-dependent heat transfer coefficients $h(S_i,t)$, i = 5 and 6, are regarded as being unknown, but everything else in eqn (1) is known. In addition, the simulated temperature readings using infrared thermography on the fin surfaces S_5 and S_6 are assumed to be available.

Let the temperature reading taken by infrared scanners on fin surfaces S_5 and S_6 be denoted by $\mathbf{Y}(S_i,t) \equiv Y(x_m,y_m,t) \equiv Y_m(S_i,t)$, m = 1 to M and i = 5 and 6, where M represents the number of measured temperatures at the extracting points. This inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y_m(S_i,t)$, estimate the unknown local time-dependent heat transfer coefficients $h(S_i,t)$.

The solution of this inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[h(S_i,t)]=\int_{t=0}^{t_f}\sum_{m=1}^{M}[T_m(S_i,t)-Y_m(S_i,t)]^2\,dt\ ;\ i=5 \text{ and } 6 \tag{2}$$

where $T_m(S_i,t)$ are the estimated or computed temperatures at the measured temperature extracting locations (x_m,y_m) and at time t. These quantities are determined from the solution of the direct problem given previously by using the estimated local heat transfer coefficients $h(S_i,t)$.

4. STEEPEST DESCENT METHOD FOR MINIMIZATION

An iterative process based on the steepest descent method [1] is now applied for the estimation of unknown heat transfer coefficients $h(S_i,t)$ by minimizing the functional $J[h(S_i,t)]$, namely

$$h^{n+1}(S_i,t)=h^n(S_i,t)-\beta^n P^n(S_i,t)\ ;\ \text{for } n=0,1,2\ldots \text{ and } i=5 \text{ and } 6 \tag{3}$$

β^n is the search step size in going from iteration n to iteration n+1, and $P^n(S_i,t)$ is the direction of descent (i.e. search direction) given by

$$P^n(S_i,t)=J'^n(S_i,t)\ ;\ i=5 \text{ and } 6 \tag{4}$$

which is the gradient direction $J'^n(S_i,t)$ at iteration n.

To complete the iterations in accordance with eqn (3), the step size β^n and the gradient of the functional $J'^n(S_i,t)$ need be computed. In order to develop expressions for determining these two quantities, a "sensitivity problem" and an "adjoint problem" need be constructed as described below.

4.1 Sensitivity problem and search step size

It is assumed that when $h(S_i,t)$ undergoes a variation Δh, T is perturbed to $T+\Delta T$. Then replacing in the direct problem h by $h+\Delta h$ and T by $T+\Delta T$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity function ΔT is obtained:

$$k\left[\frac{\partial^2 \Delta T(\Omega,t)}{\partial x^2}+\frac{\partial^2 \Delta T(\Omega,t)}{\partial y^2}+\frac{\partial^2 \Delta T(\Omega,t)}{\partial z^2}\right]=\rho Cp\frac{\partial \Delta T(\Omega,t)}{\partial t}\ ;\ \text{in}\ \Omega(x,y,z,t)\ ,\ t>0 \qquad (5a)$$

$$\frac{\partial \Delta T(S_i,t)}{\partial n}=0\quad ;\ \text{on fin surfaces}\ S_i\ ,\ i=1\ \text{to}\ 4\ ,\ t>0 \qquad (5b)$$

$$-h(S_5,t)\Delta T+k\frac{\partial \Delta T}{\partial z}=\Delta h(S_5,t)(T-T_\infty)\quad ;\ \text{on fin surface}\ S_5\ ,\ t>0 \qquad (5c)$$

$$h(S_6,t)\Delta T+k\frac{\partial \Delta T}{\partial z}=\Delta h(S_6,t)(T_\infty-T)\quad ;\ \text{on fin surface}\ S_6\ ,\ t>0 \qquad (5d)$$

$$\Delta T(S_i,t)=0\quad ;\ \text{on tube surfaces}\ ,\ i=7\ \text{to}\ I+6,\ t>0 \qquad (5e)$$

$$\Delta T(\Omega,t)=0\quad ;\ \text{for}\ t=0 \qquad (5f)$$

By following the standard process as described in [8], the search step size β^n can be determined as:

$$\beta^n=\frac{\int_{t=0}^{t_f}\sum_{m=1}^{M}[T_m(S_i,t)-Y_m(S_i,t)]\Delta T_m(S_i,t)dt}{\int_{t=0}^{t_f}\sum_{m=1}^{M}[\Delta T_m(S_i,t)]^2 dt}\ ;\ i=5\ \text{and}\ 6 \qquad (6)$$

4.2 Adjoint problem and gradient equation

To obtain the adjoint problem, eqn (1a) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda(\Omega,t)$ and the resulting expression is integrated over the correspondent space domain. Then the result is added to the right hand side of eqn (2). By following the standard process as described in [8], the following adjoint problem for the determination of $\lambda(\Omega,t)$ can be obtained:

$$k\left[\frac{\partial^2 \lambda(\Omega,t)}{\partial x^2}+\frac{\partial^2 \lambda(\Omega,t)}{\partial y^2}+\frac{\partial^2 \lambda(\Omega,t)}{\partial z^2}\right]+\rho Cp\frac{\partial \lambda(\Omega,t)}{\partial t}=0\ ;\ \text{in}\ (\Omega,t)\ ,\ t>0 \qquad (7a)$$

$$\frac{\partial \lambda(\Omega,t)}{\partial n}=0\ ;\ \text{on fin surfaces}\ S_i\ ,\ i=1\ \text{to}\ 4\ ,\ t>0 \qquad (7b)$$

$$-\lambda h+k\frac{\partial \lambda}{\partial n}=2k[T(S_5,t)-Y(S_5,t)]\delta(x-x_m)\delta(y-y_m)\ ;\ \text{on fin surface}\ S_5\ ,\ t>0 \qquad (7c)$$

$$\lambda h+k\frac{\partial \lambda}{\partial n}=2k[T(S_6,t)-Y(S_6,t)]\delta(x-x_m)\delta(y-y_m)\ ;\ \text{on fin surface}\ S_6\ ,\ t>0 \qquad (7d)$$

$\lambda(S_i, t) = 0$; on tube surfaces , i = 7 to I+6 , t > 0 (7e)

$\lambda(\Omega,t) = 0$; for $t = t_f$ (7f)

Finally, the gradient of the functional $J[h(S_i,t]$ can be obtained as:

$$J'[h(S_i,t)] = \frac{\lambda(S_i,t)}{k}[T(S_i,t) - T_\infty] \text{ ; on surfaces } S_i \text{ , i = 5 and 6} \quad (8)$$

5. RESULTS AND DISCUSSIONS

The objective of this study is to show the validity of the SDM in estimating the time-dependent local surface heat transfer coefficients for a 3-D plate finned-tube heat exchangers with no prior information on the functional form of the unknown function. The physical model for this problem is described as follows: The thermal conductivity for the plate fin is taken as k = 20W/(m-K), ρ = 7850 kg/m^3, Cp = 440 J/kg-K; ambient temperature is chosen as T_∞ = 296 K and the temperatures on all tube surfaces are assumed as $T(S_i,t)$ = 353 K, i = 7 to (I+6).

One of the advantages of using the SDM is that the initial guesses of the unknown heat transfer coefficients $h(S_i,t)$ can be chosen arbitrarily. In all the test cases considered here, the initial guesses for heat transfer coefficients used to begin the iteration are taken as $h(S_i,t) = 0.0$.

In order to compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data **Y** can be expressed as

$$Y_m = Y_{m,exact} + \omega\, \sigma \quad (9)$$

where $Y_{m,exact}$ is the solution of the direct problem with exact heat transfer coefficients; σ is the standard deviation of the measurements; and ω is a random variable, that generated by subroutine DRNNOR of the IMSL [8], is within -2.576 to 2.576 for a 99% confidence bound. In order to simplify the problem, the measurement errors on the surfaces S_5 and S_6 are assumed the same.

We now present below the numerical experiments in determining $h(S_i,t)$ using the inverse analysis. The geometry and grid system for the first test case, i.e. staggered tube arrangement for a fin plate, are shown in Figures 2(a) and 3(a), respectively. The dimensions for the fin in the x, y and z directions are 220 mm, 170 mm and 1mm, respectively. The radius of the tube is taken as 12.7 mm and the longitudinal pitch of the tube, i.e the distance between center of the two tubes, is 60.7 mm. The number of grids in the z-direction is taken as 5 and the total grid number on the x-y plane is 1456. The measured temperature extracting locations are at the grid points. The measurement time period Δt is 150 seconds and the total measurement time t_f is 3750 seconds, i.e. there are 25 time steps. Therefore there exist a total of 36400 unknown discrete heat transfer coefficients in this study.

The simulated exact function of the surface heat transfer coefficients on surfaces S_5 and S_6 in this numerical experiment is assigned in the following manner: (a) Firstly, solve eqn (1a) by assuming the following boundary and initial conditions:

$$T(S_3,t) = 20 + \frac{(y_{max} - y)}{y_{max}} \times 69 \text{ ; on } S_3\text{, where } y_{max} = 220 \text{ , } t > 0 \quad (10a)$$

$$T(S_4,t) = 20 \quad \text{; on } S_4 \text{ , } t > 0 \quad (10b)$$

$$\frac{\partial T(S_i,t)}{\partial n} = 0 \quad \text{; on the rest of surfaces , } t > 0 \quad (10c)$$

$$T(\Omega, t) = 20 \quad \text{; at } t = 0 \quad (10d)$$

(b) Secondly, the values of the calculated temperature distributions on S_5 and S_6 are then taken as the simulated exact heat transfer coefficients.

The three-dimensional inverse problem is first examined by using exact measurements, i.e. σ = 0.0. After 30 iterations the inverse solutions converged. The exact and estimated (or calculated) heat transfer coefficients $h(S_5,t)$ at time t = 3600 s are reported in Figure 2.

The estimated heat transfer coefficients are also close to the exact values. The relative error between exact and estimated heat transfer coefficients is calculated as ERR1 = 2.92 %, where ERR1 is defined as

$$\text{ERR1 \%} = \left[\sum_{j=1}^{J} \sum_{m=1}^{M} \left| \frac{h_m(S_5,j) - \hat{h}_m(S_5,j)}{h_m(S_5,j)} \right| \right] \div (M \times J) \times 100\,\% \quad (11)$$

where J represents the number of the discreted times, M the number of grids and $\hat{h}_{m,j}(S_5)$ the estimated values.

The corresponding measured and estimated temperature distributions at time t = 3600 s are shown in Figure 3. By comparing Figures 3(a) and 3(b) we find that the estimated temperatures are almost identical to the measured temperatures since the relative error between the measured and calculated temperatures is calculated as ERR2 = 0.025%, where ERR2 is defined as

$$\text{ERR2 \%} = \left[\sum_{j=1}^{J} \sum_{m=1}^{M} \left| \frac{T_m(S_5, j) - Y_m(S_5, j)}{Y_m(S_5, j)} \right| \right] \div (M \times J) \times 100\,\% \tag{12}$$

The inverse calculation then proceeds to consider the inexact temperature measurements. The standard deviation of the measurements is first taken as $\sigma = 0.1$, then it was increased to $\sigma = 0.3$.

For $\sigma = 0.1$, 10 iterations are needed to satisfy the stopping criteria based on the discrepancy principle, the estimated heat transfer coefficients at times t = 2250 s and 3600 s are shown in Figure 4. The relative errors for the heat transfer coefficients and the temperatures are calculated as ERR1 = 7.80 % and ERR2 = 0.036 %. For $\sigma = 0.3$, the number of iterations to satisfy the stopping criterion is only 8, the estimated heat transfer coefficients at times t = 2250 s and 3600 s are shown in Figure 5, and the relative errors for the heat transfer coefficients and the temperatures are calculated as ERR1 = 11.6 % and ERR2 = 0.068 %. Based on the above numerical results, we concluded that the estimated heat transfer coefficients are sensitive to the measurement errors, and therefore for this reason an accurate measurement technique is required for such kind of problem.

6. CONCLUSIONS

The SDM with an adjoint equation was successfully applied in determining the time-dependent local heat transfer coefficients for plate finned-tube heat exchangers for a 3-D inverse heat conduction problem. Two test cases involving different arrangement of fins, different type of heat transfer coefficients and different measurement errors were considered. The results show that the SDM does not require a priori information for the functional form of the unknown functions and reliable estimated values can always be obtained.

Acknowledgment

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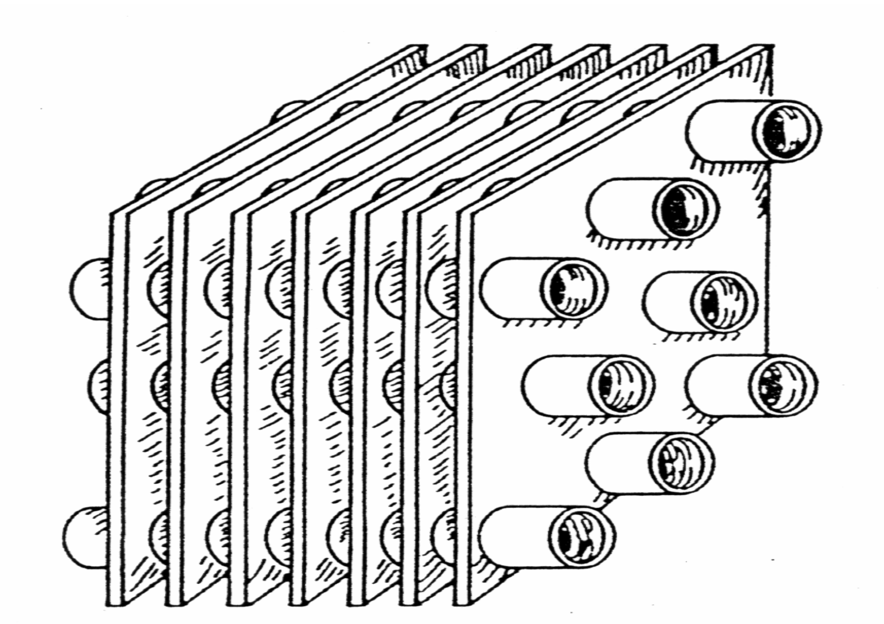


Figure 1(a). A typical plate finned-tube heat exchanger.

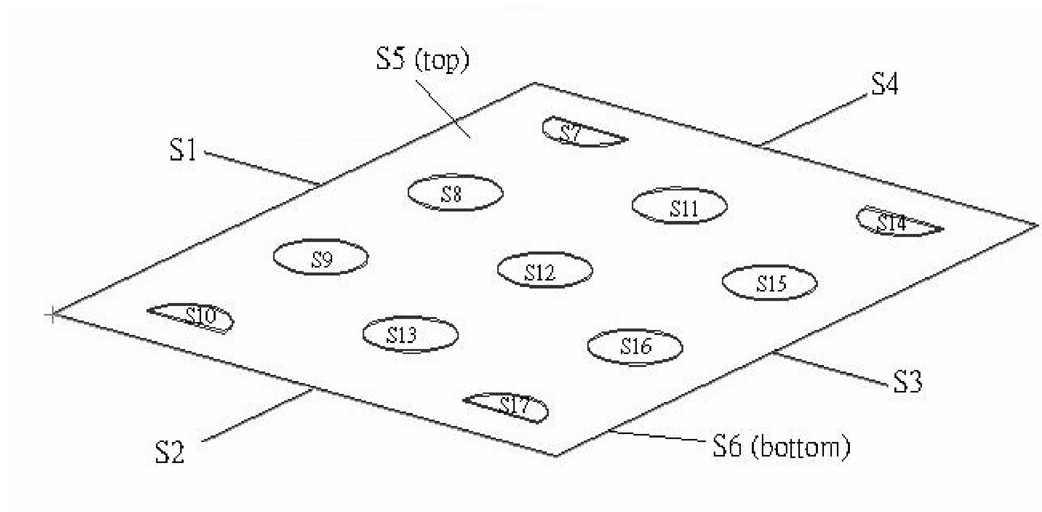
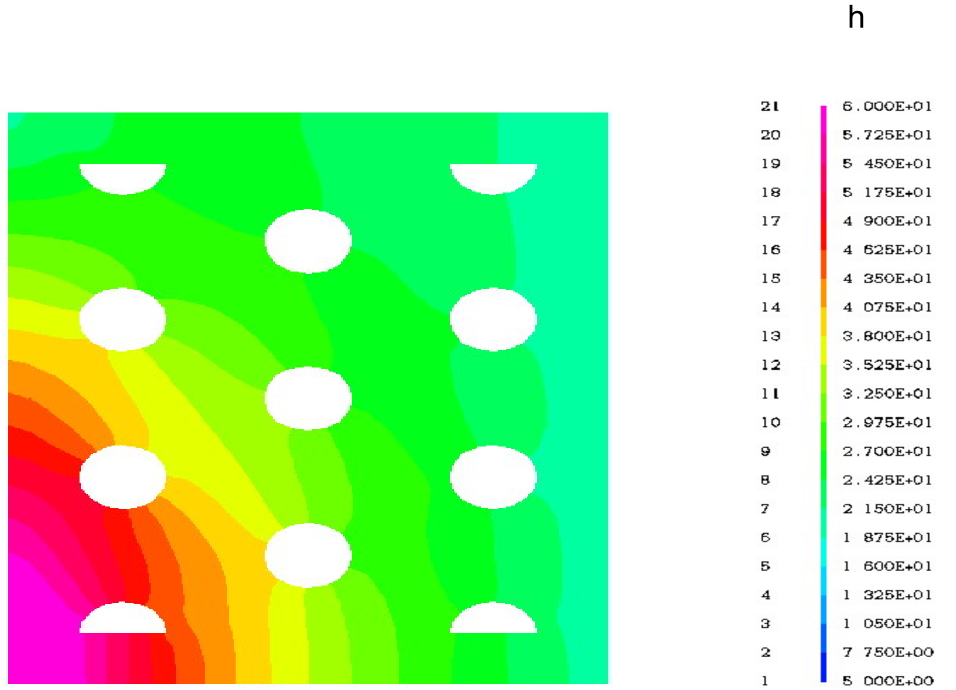
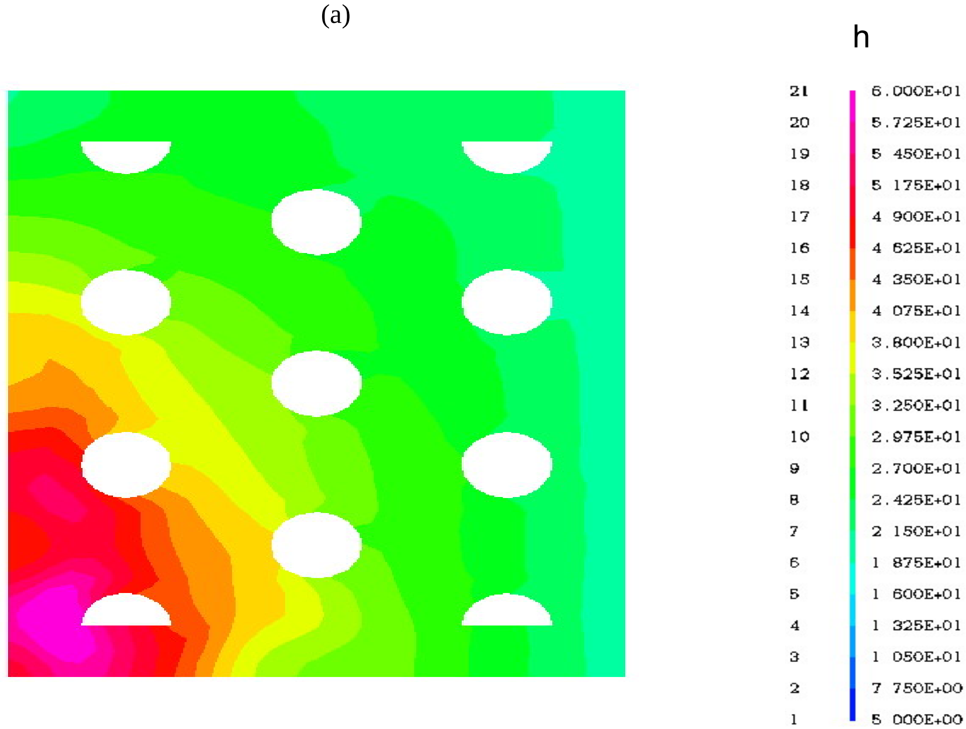


Figure 1(b). The geometry of plate fin in staggered arrangement.

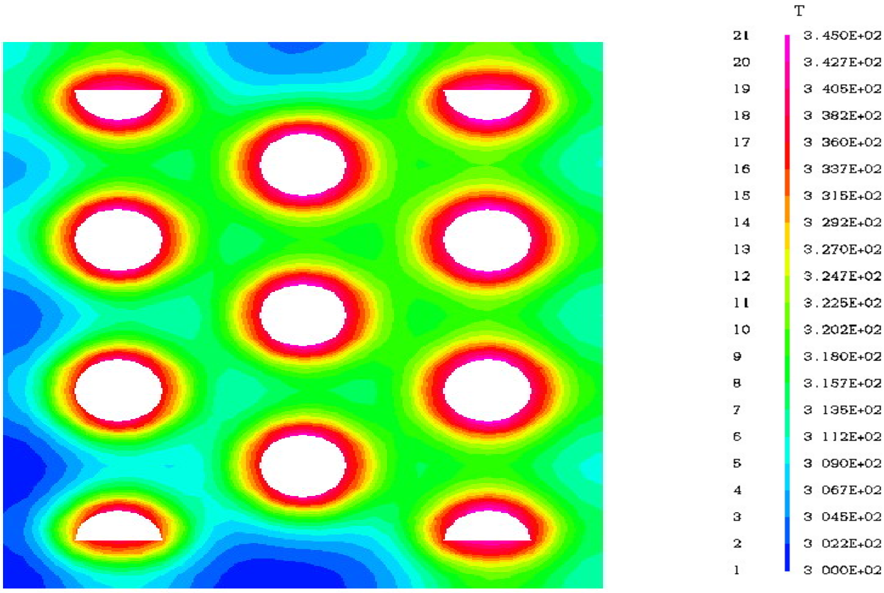


(a)

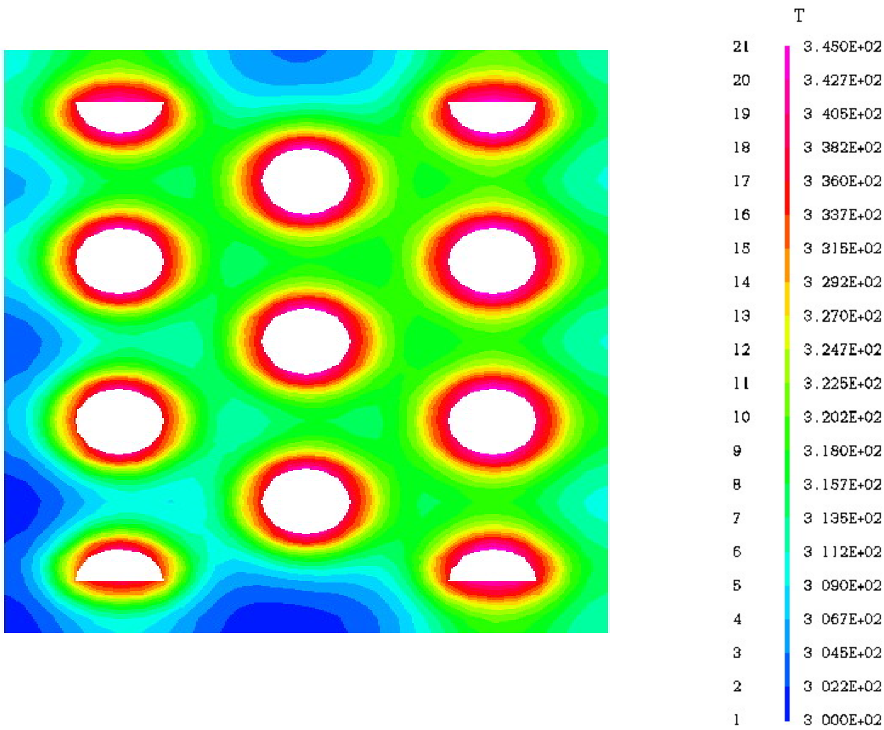


(b)

Figure 2. (a) The exact, and (b) estimated, heat transfer coefficients at t = 3600 s with $\sigma = 0.0$.

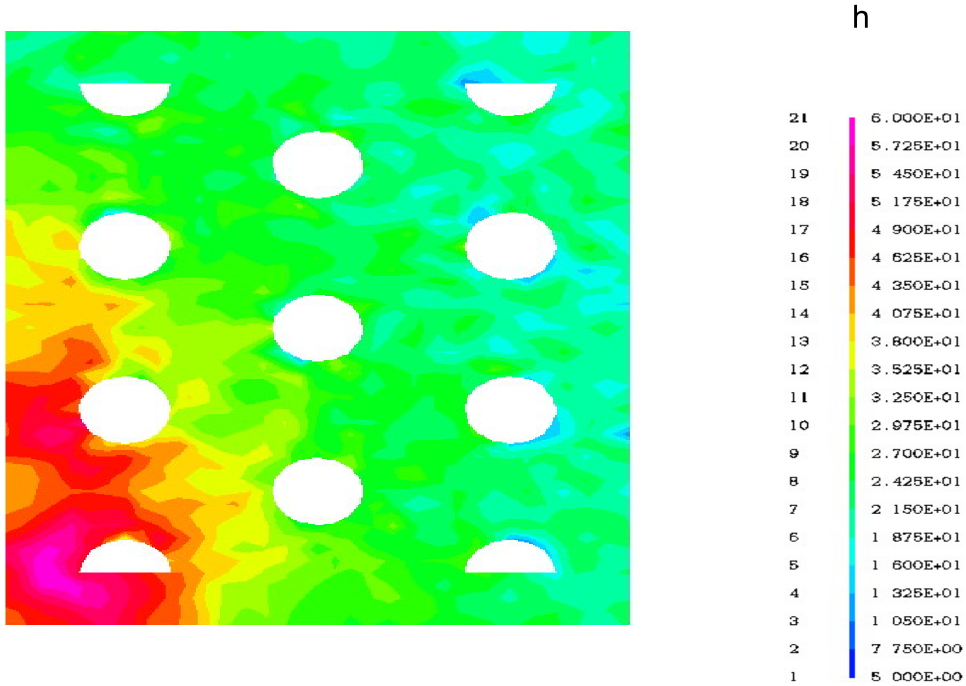


(a)

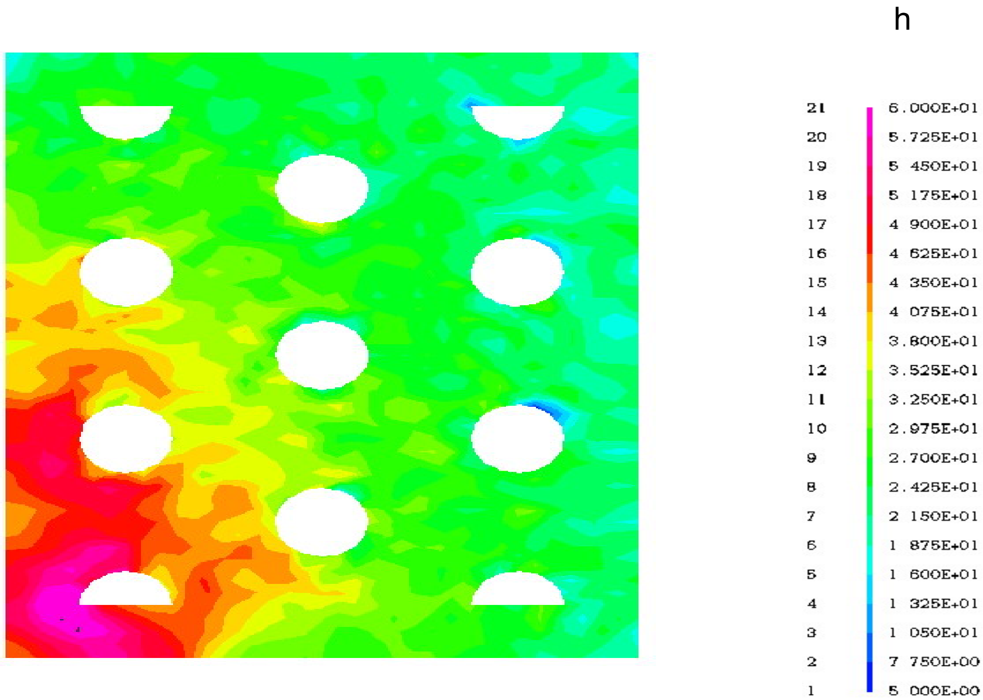


(b)

Figure 3. (a) The measured, and (b) estimated, temperatures at t = 3600 s with $\sigma = 0.0$.

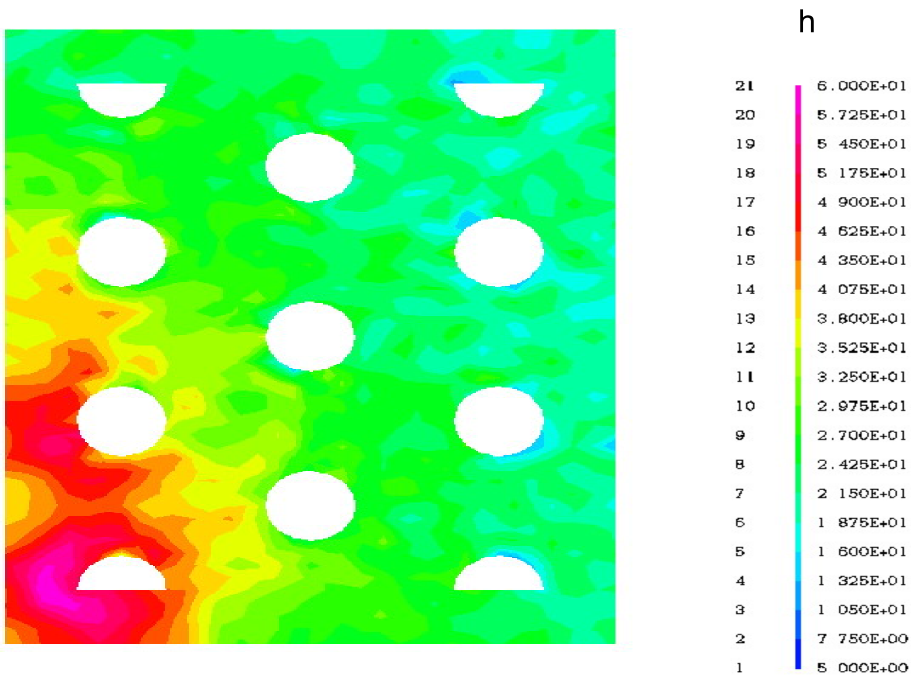


(a)

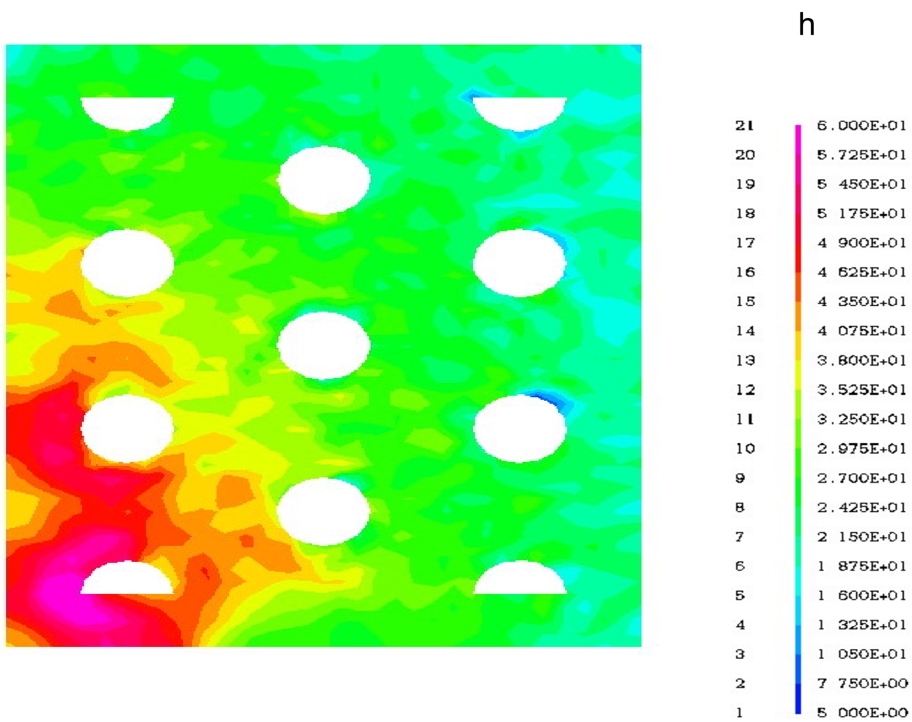


(b)

Figure 4. The estimated heat transfer coefficients at (a) $t = 2250$ s, and (b) $t = 3600$ s, with $\sigma = 0.1$.



(a)



(b)

Figure 5. The estimated heat transfer coefficients at (a) $t = 2250$ s, and (b) $t = 3600$ s, with $\sigma = 0.3$.